Improvement of pedestrian flow by slow rhythm

Daichi Yanagisawa,[1,*] Akiyasu Tomoeda,[2,3] and Katsuhiro Nishinari[4]
[1]*College of Science, Ibaraki University, Ibaraki 310-8512, Japan*
[2]*Meiji Institute for Advanced Study of Mathematical Sciences, Meiji Univeristy, Kanagawa 214-8571, Japan*
[3]*CREST, Japan Science and Technology Agency, Kanagawa 214-8571, Japan*
[4]*Research Center for Advanced Science and Technology, The University of Tokyo, Tokyo 153-8904, Japan*
(Received 28 August 2011; published 23 January 2012)

We have developed a simple model for pedestrians by dividing walking velocity into two parts, which are *step size* and *pace of walking* (number of steps per unit time). Theoretical analysis on pace indicates that rhythm that is *slower* than normal-walking pace in a low-density regime increases flow if the flow-density diagram is convex downward in a high-density regime. In order to verify this result, we have performed an experiment with real pedestrians and observed the improvement of flow in a congested situation using slow rhythm.

DOI: 10.1103/PhysRevE.85.016111 PACS number(s): 05.65.+b, 45.70.Vn, 05.90.+m, 64.60.−i

I. INTRODUCTION

Dynamics of pedestrians, as well as that of other self-driven particles [1], has been vigorously studied by physicists over recent decades since it is an interesting collective phenomenon in nonequilibrium physics, and formation of jam is considered to be a dynamical phase transition. Many pedestrian-dynamics phenomena, such as arching and lane formation, are analyzed by macroscopic and microscopic models [2]. Flow-density diagrams (FDDs) and velocity-density diagrams, which are often called fundamental diagrams (FDs), are also investigated actively by both simulation [3] and experiment [4] to reveal the basic characteristics of pedestrian dynamics. In addition to the existence of free-flow and jam phases, the linear relation between velocity and headway and the subsequent transition to *turbulent* flow are observed in Refs. [5] and [6], respectively. Influence of experimental and observational conditions on FDs is also being investigated diligently. Reference [7] has clarified how different measurement methods and configurations affect FDs, and Ref. [8] has investigated an international difference between FDs. Furthermore, a model that reproduces the FDs in several geometries is being developed [9].

Elucidation of collective phenomena and the FD in pedestrian dynamics is an important mission for physicists engaged in the research of self-driven particles. Besides, development of solutions to ease congestion and contribution to safety [10] are also expected, since many pedestrians suffer from congestion in large cities and from difficulty in disaster evacuations all over the world. However, the details of pedestrian FDs have not been understood well enough to develop a general solution to pedestrian congestion, so that few concrete methods have been presented so far in spite of the many successful studies of pedestrian dynamics. Therefore, in this paper we focus on deriving a method to improve pedestrian flow in one of the simplest situations rather than pursuing an accurate FD by considering many factors.

In Ref. [11] the effect of music on an individual pedestrians has been studied experimentally. Inspired by this research, we analyze the effect of rhythm on crowded pedestrians theoretically by our simple model and reveal that *slow* rhythm increases pedestrian flow in congested situations without any danger. The result is also verified by our real experiments.

II. MODEL AND THEORETICAL ANALYSIS

A. Assumption of homogeneous distribution

We consider a one-dimensional periodic circuit whose length is L. The width of the circuit is as wide as that of one pedestrian, so that pedestrians walk in a line without overtaking. Most FDs depict quasi-one-dimensional flow, where overtaking is acceptable, and the variety of FDs partially arises from overtaking. Thus, this condition contributes to reduction of variety and focuses on a simple situation. A similar pedestrian flow is studied in Refs. [5] and [12]. Moreover, we assume that N homogeneous pedestrians, whose length is b, distribute *homogeneously* in the circuit without considering complex microscopic interaction. Thus, the pedestrians in our model have average properties of real pedestrians, and the model does not reproduce important phenomena such as the propagation of stop waves in the congested situation; however, it still preserves the excluded volume of pedestrians and enables us to obtain a method for increasing flow.

Due to the homogeneous spatial distribution of pedestrians, the density ρ and headway h are calculated as

$$\rho = \frac{N}{L}, \qquad h = \frac{L - bN}{N} \in [0,\infty), \tag{1}$$

respectively. Thus, the relation between ρ and h is described by

$$\rho = \frac{1}{b+h}, \qquad h = \frac{1 - b\rho}{\rho}. \tag{2}$$

From these equations, the range of ρ is $(0,1/b]$.

---

*daichi@mx.ibaraki.ac.jp; http://yana.sci.ibaraki.ac.jp/

B. Step-size function

Since we have simplified the spatial distribution of pedestrians, we consider the velocity V of an individual in detail by dividing it into two parts,

$$V(\rho) = S(\rho)P(\rho), \tag{3}$$

where S (step-size function) and P (pace function) denote step size and pace of walking (total number of right and left steps per unit time), respectively. The figures in Ref. [11] indicate that there is not a strong correlation between step size and pace if pace is not extremely high, i.e., lower than about 120 BPM (beats per minute), and the velocity linearly increases as pace increases. Thus, this formulation is experimentally justified if we consider pedestrians walking without haste.

The explicit formulation of the step-size function is intuitively determined as follows. It is plausible to assume that there is maximum step size for pedestrians, which is given as s, in the low-density regime. When the density becomes large, pedestrians are no longer able to walk with their maximum step size. Since we assume homogeneous spatial distribution, the pedestrians can maximally proceed with headway h in one step; however the effect of personal space, which is usually larger than the size of the pedestrians in the direction of motion b, prevents the pedestrians from contacting their predecessors. This phenomenon is introduced by the parameter $k \in (0,1]$, and the step-size function is

$$S(\rho) = \begin{cases} s & (0 \leqslant \rho \leqslant \rho_c), \\ kh(\rho) & (\rho_c < \rho \leqslant \rho_j), \end{cases} \tag{4}$$

where $\rho_c = k/(kb+s)$ is calculated from the equation $kh(\rho_c) = s$, and $\rho_j = 1/b$ is the maximum density. Because of the homogeneity of pedestrians and their spatial distribution, the flow Q is computed by using the hydrodynamic equation and individual velocity V as $Q = \rho V$. The characteristics of the step-size function coincide with the result in Ref. [5] that the velocity linearly increases as the headway increases.

C. Pace function

Now let us start discussion of the pace function. If the density is low and pedestrians do not interact with each other, it is feasible to assume that pedestrians walk with constant pace. However, contrary to the step-size function, it is difficult to obtain the explicit formulation of the pace function in the high-density regime with some intuitive assumptions. Thus, we consider a simple linear function,

$$P(\rho) = \begin{cases} p & (0 \leqslant \rho \leqslant \rho_c), \\ p - a[h_c - h(\rho)] & (\rho_c < \rho \leqslant \rho_j), \end{cases} \tag{5}$$

where $h_c = h(\rho_c)$, and investigate how the change of pace affects the flow. The parameter p represents pace in the free-flow situation and a represents the influence of headway and density on pace. If $a > 0$ ($a < 0$) the pace of pedestrians decreases (increases) when the density increases, and if $a = 0$ pedestrians keep walking with constant pace p regardless of the headway. Note that $a \leqslant p/h_c = kp/s$ since $P(\rho) \geqslant 0$. We assume that pedestrians start to affect their followers when the density becomes larger than ρ_c, so that the formulations of both the step-size and pace functions are transformed there. This unification of the critical point also contributes to the simplicity of our model.

By considering Eqs. (4), (5), and the hydrodynamic equation, the explicit mathematical formulation of the flow Q is obtained as

$$Q(\rho) = \begin{cases} sp\rho & (0 \leqslant \rho \leqslant \rho_c), \\ k(1-b\rho)\left[p - a\left(\frac{1-b\rho_c}{\rho_c} - \frac{1-b\rho}{\rho}\right)\right] & (\rho_c < \rho \leqslant \rho_j). \end{cases} \tag{6}$$

The maximum of the flow is calculated as

$$Q_{\max} = \begin{cases} sp\rho_c & (a \geqslant a_c), \\ kp_j - 2kab(1 - \sqrt{1 - p_j/(ab)}) & (a < a_c), \end{cases} \tag{7}$$

which is achieved at

$$\rho = \begin{cases} \rho_c, \\ [b\sqrt{1 - p_j/(ab)}]^{-1}, \end{cases} \tag{8}$$

respectively. The parameters p_j and a_c are described as follows:

$$p_j = P(\rho_j) = p - ah_c, \qquad a_c = -\frac{bp}{h_c(b+h_c)}. \tag{9}$$

As we can see from the expressions above, the density where the maximum flow is attained is strongly influenced by the parameter a.

The second derivative of the flow,

$$\frac{d^2Q(\rho)}{d\rho^2} = \frac{1}{\rho^3}\frac{d^2V(h)}{dh^2} = \begin{cases} 0 & (0 < \rho < \rho_c) \\ 2ka/\rho^3, & (\rho_c < \rho < \rho_j), \end{cases} \tag{10}$$

indicates that convexity of the flow Q in the high-density regime is dominated by the parameter a. Therefore, the parameter a, which controls the pace in the high-density regime, plays important roles in our model. The plots of the flow Q for three kinds of a are shown in Fig. 1. We see that the FDD greatly varies according to the parameter a.

D. Improvement of flow by slow rhythm

Here we would like to propose a solution to improve flow in the congested situation from our model, in which the velocity of pedestrians is represented by the product of step size (4) and pace (5).

If pedestrians walk with a constant rhythm, in other words if we can control the walking pace by rhythm using a device such as a metronome, the rhythm exactly corresponds to the pace, i.e., the parameter p. Therefore, fast and slow rhythms increase and decrease the flow, respectively, as we can verify from Eq. (6). In this case, the parameter $a = 0$ because pedestrians walk with constant pace p^R irrespective of the density.

We denote the pace of walking without rhythm, i.e., normal walking, in the free-flow situation as p^N, and assume that it decreases in the high-density regime, i.e., $a = a^N > 0$, which seems more realistic than $a < 0$. Then we surprisingly observe the crossing between the two curves corresponding to normal walking $(p,a) = (1,0.5)$ and *slow*-rhythmic walking $(p,a) = (0.8,0)$ seen in Fig. 1. Due to the convexities of

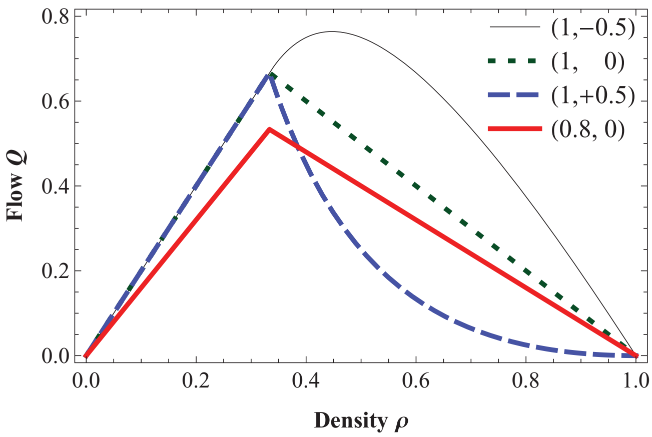

FIG. 1. (Color online) Theoretical FDDs for various (p,a) in the case $b = 1$, $s = 2$, and $k = 1$. We observe the crossing between the curves (1,0.5) and (0.8,0), which correspond to normal and slow-rhythmic walking, respectively, at $\rho_s = 5/13$. Note that the curves are not smooth at $\rho_c = 1/3$ due to the definition of our model Eqs. (4)–(6).

the FDDs in the normal and rhythmic walking cases, i.e., convex downward and linear, a rhythm that is *slower* than the normal-walking pace increases the flow in the high-density regime. The crossing is achieved at

$$\rho_s = \rho_c \left[1 - \frac{p^N - p^R}{a} \rho_c \right]^{-1}, \tag{11}$$

where $\rho_c < \rho_s < \rho_j$, if the condition

$$p_j^N (= p^N - a^N h_c) < p^R < p^N \tag{12}$$

is satisfied.

This phenomenon may give a solution to ease congestion in the real world. The significant advantage is that any excessive haste is not necessary at all. The flow increases by just keeping a slow walking pace. Thus, pedestrians do not consume extra energy or conflict with others by moving aggressively in pedestrian jams.

III. EXPERIMENT

A. Experimental setup

We have performed an experiment with real pedestrians to verify the theoretical result obtained in Sec. II D. We constructed a circuit whose inner and outer radii were $r_i = 1.8$ m and $r_o = 2.3$ m, respectively. The participants of the experiment, who were male university students between eighteen and thirty-nine years old, walked the circuit in the counter-clockwise direction. In the beginning of each trial we briefly instructed participants to distribute homogeneously in the circuit without signs on the floor or measuring the distance between each participant. A snapshot of the experiment is shown in Fig. 2.

We executed eleven kinds of density conditions. The number of the participants in the circuit in each condition was $N = \{1, 3, 6, 9, \ldots, 30\}$. The conditions $N = 1$ and $N = 3$ were tried three times with different participants, and the other

FIG. 2. (Color online) Snapshot of the experiment. Normal walking, $N = 15$ ($\rho = 1.16$ persons/m).

conditions were tried once. Each trial was more than one minute in the cases $N \geqslant 3$. The global density is calculated as

$$\rho = \frac{N}{L} = \frac{N}{\pi\,(r_i + r_o)} \tag{13}$$

and was used to depict FDDs.

Two kinds of walking were performed in the experiment. In the first case, we did not give any specific instructions to the participants, so that they walked normally. In the latter case, the participants were instructed to walk with the sound from the electric metronome, whose rhythm is 70 BPM. Note that we did not inform which foot to move first.

In the case $N = 1$, we measured the lap time for completeing a circuit. In the case $N \geqslant 3$, we measured the time that each participant passes the measuring point in the circuit and depicted the cumulative plots, which show the evolution of the total number of participants who passed the measuring point. Then, linear regression analysis gives pedestrian flows as the slope of the cumulative plots.

B. Experimental verification of the effect of rhythm

Figure 3 shows the FDDs obtained from our model and experiment, which quantitatively correspond with each other very well. From the figure, first we see that the flow is larger in the normal case than the rhythmic case in the low-density regime. Hence, the pace 70 BPM is much slower than the normal-walking pace of the participants, and the flow becomes smaller if the participants try to walk with the slow rhythm. Second, linearity of the flows in the low-density regime verified that participants walked with constant step size and pace. Third, the flow decreases linearly in the rhythmic case as expected from our analysis. This result is compatible with the fact that step size changes as in Eq. (4). Furthermore, we observe that the flow in the normal case is convex downward in the high-density regime, as we have assumed in the theoretical analysis. Thus, the walking pace decreases from the influence of the predecessors. We consider that the clear convexity, which was not seen in the previous experiment in Ref. [5], is observed because we performed the experiment with a density of more than 2.0 persons/m. Finally, since the theoretical assumptions of the convexity are satisfied in the experimental flows, we observe the crossing of the two plots. In other words, the flow of the rhythmic case exceeds that of the normal case

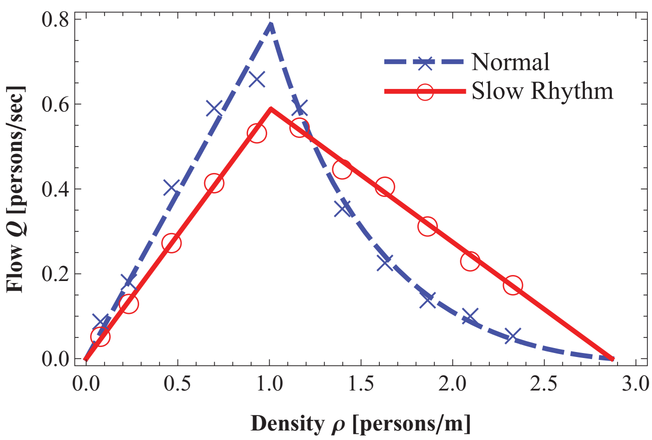

FIG. 3. (Color online) FDD of normal walking and rhythmic walking (70 BPM). The curves and markers represent the theoretical and experimental flows, respectively. The experimental values at $\rho = 0.08$ and 0.23 persons/m, i.e., $N = 1$ and 3, respectively, are averages of the three trials. The parameters $b = 0.35$ m, $s = 0.5$ m, $k = 0.78$, $p^N = 1.56$ BPS (beats per second) = 94 BPM, and $a = 2.2$ BPS m are obtained by the least-squares approach with the experimental data. We clearly observe the crossing, and slow-rhythmic walking achieves larger flow in the high-density regime.

in the high-density regime. Therefore, we have succeeded in verifying that slow rhythm improves the pedestrian flow.

C. Analysis of errors

The standard error of each flow (slope) is shown in Fig. 4. Their orders are 10^{-3} and are much smaller than orders of the mean flows (10^{-1}). Moreover, the determination coefficients are larger than 0.99 in all the conditions. Thus, the participants passed the measuring point rather constantly, and the assumption of homogeneous spatial distribution is not greatly harmed in our experiment. Due to the small inertia of pedestrians, clear visibility, and small size of the circuit, the participants did not form large clusters in the congested situation, so that the flows calculated from our model quantitatively agree well with the experimental flows in spite of the very simple model.

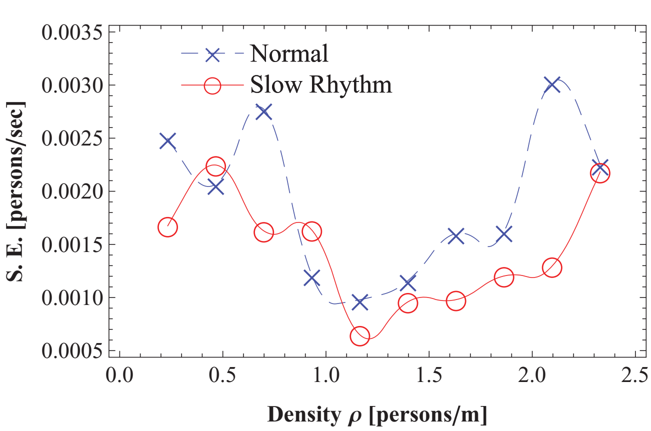

FIG. 4. (Color online) Standard errors of the experimental flows by linear regression. The curves are added to improve visibility.

Although the errors are small, we observe that they change according to the density. In the low-density regime, the errors are large since the participants easily acquire headway that is larger than the maximum step size s without homogeneous spatial distribution. A procession, whose head is the slowest participant, is formed, and some vacant space is observed after the tail participant. In the middle-density regime, the errors decrease since the participants' effort to acquire headway results in homogeneous spatial distribution. In the highdensity regime, the errors become large again. This is because some participants move promptly with small headway, about 30 cm, while the others do not move and wait for enough headway for one large step. Variety in the size of personal space results in the larger errors in the high-density regime than in the middle-density regime. Note that the heterogeneity of the spatial distribution and the participants discussed in this paragraph does not greatly harm our results, as we have seen in the previous sections.

The errors are smaller in the rhythmic-walking case than in the normal-walking case in the high-density regime. The reversal of the flow, which is the main result in this paper, comes from the retention of the walking pace using rhythm. On the other hand this result implies that rhythm removes the heterogeneity of pedestrians' movement, synchronizes it, and contributes to the homogeneous spatial distribution. This synchronization effect by rhythm should be studied in detail in the near future.

IV. CONCLUDING DISCUSSION

In this paper, we have obtained the FDD of pedestrians by dividing the velocity term into the step-size and pace parts. Analyzing the effect of pace, we have surprisingly discovered that a rhythm that is slower than the normal-walking pace in the free-flow situation increases the flow in the congested situation. In spite of the assumption that both characteristics and spatial distribution of pedestrians are homogeneous, the result of our model agrees very well with that of the experiment, and the improvement of flow is experimentally verified. The analysis of errors indicates that rhythm also contributes to synchronize movement of pedestrians.

We have succeeded in presenting a method to increase flow in a simple situation, and more generalized research will enhance its practical utility.

Our qualitative result, i.e., improvement of flow by slow rhythm, mainly depends on the two characteristics of pedestrians. The first one is that pedestrians avoid conflict with their predecessors by decreasing step size and pace in the congested situation, and the other one is that they can walk with the rhythm. Thus, we consider that same phenomenon will be observed if most participants of the experiment share the same characteristics above. However, the quantitative results, such as the value of density where the crossing of the normal and rhythmic flows is achieved, will be changed according to participants of the experiment. Therefore, the dependence of the parameters on the characteristics of pedestrians and the geometry should be studied in detail. The step size s and the pace p obtained in the normal-walking case in our experiment are smaller than those in Ref. [11]. This may be attributed to the difference of the body length of the participants and the configuration of the circuit. The change of pace against density has not been researched so far, so more experimental data are required to investigate how the parameter a varies with the conditions. In the case where pedestrians move fast, such as evacuation, the step-size function needs to be extended since personal space, whose effect is represented by the constant parameter k, varies with velocity [9].

These parameters are determined as the representative values of the participants of our experiment by the least-squares approach. If we obtain the individual parameters of the participants, it is possible to analyze how they become the parameters of a group. Then we can suggest the most effective approach to smooth pedestrian flow in the congested situation based on the information of individuals.

Since our model is simple, we are able to extend it by introducing the delay of reaction and the probabilistic factors from the previous sophisticated models for traffic and pedestrian dynamics [1]. This extension will enable us to study microscopic interaction between pedestrians and the deviation of the flow (Fig. 4) in the rhythmic-walking case.

In our experiment, we have instructed the participants to walk with the rhythm from the electronic metronome; however, it is not certain whether pedestrians in the real world would walk with the rhythm as in the experiment. Moreover, we have observed that there were a few participants who did not clearly walk with the rhythm, although the influence from them is negligible. The same kind of examinees are also reported in Ref. [11]. Therefore, it is significant to study the relation between the improvement of flow and the ratio of pedestrians walking with the rhythm. Investigation of the way to synchronize pedestrians' movement to the rhythm without explicit instruction is another important future work to apply the rhythmic-walking method to the real world.

Researches on higher-dimensional cases are also necessary. In the quasi-one-dimensional case, where the width of the circuit is larger than one pedestrian, fast pedestrians are able to pass slow ones. When we consider bidirectional flow and two-dimensional cases, we need to consider conflicts between pedestrians. Therefore, investigation of the effect of rhythm may not be as simple as the one-dimensional case in this paper; however, success of the study broadens the range of application of the slow-rhythmic flow and contributes to achieving a smooth-flow society.

ACKNOWLEDGMENTS

The experiment was authorized by the Office for Life Science Research Ethics and Safety of The University of Tokyo. This work is financially supported by the Japan Society for the Promotion of Science and the Japan Science and Technology Agency.

[1] D. Helbing, Rev. Mod. Phys. **73**, 1067 (2001); K. Nishinari, Y. Okada, A. Schadschneider, and D. Chowdhury, Phys. Rev. Lett. **95**, 118101 (2005); A. John, A. Schadschneider, D. Chowdhury, and K. Nishinari, *ibid.* **102**, 108001 (2009); A. Schadschneider, D. Chowdhury, and K. Nishinari, *Stochastic Transport in Complex Systems* (Elsevier, Amsterdam, 2010).

[2] D. Helbing, Complex Syst. **6**, 391 (1992); D. Helbing and P. Molnár, Phys. Rev. E **51**, 4282 (1995); D. Helbing, I. Farkas, and T. Vicsek, Nature (London) **407**, 487 (2000); C. Burstedde, K. Klauck, A. Schadschneider, and J. Zittartz, Physica A **295**, 507 (2001).

[3] A. Seyfried, B. Steffen, and T. Lippert, Physica A **368**, 232 (2006); D. R. Parisi, M. Gilman, and H. Moldovan, *ibid.* **388**, 3600 (2009).

[4] Online database at ped-net.org homepage and the references therein, http://www.ped-net.org/.

[5] A. Seyfried, B. Steffen, W. Klingsch, and M. Boltes, J. Stat. Mech. (2005) P10002.

[6] D. Helbing, A. Johansson, and H. Z. Al-Abideen, Phys. Rev. E **75**, 046109 (2007).

[7] J. Zhang, W. Klingsch, A. Schadschneider, and A. Seyfried, J. Stat. Mech. (2011) P06004.

[8] U. Chattaraj, A. Seyfried, and P. Chakroborty, Adv. Complex Syst. **12**, 393 (2009); e-print arXiv:0903.0149.

[9] M. Chraibi, A. Seyfried, and A. Schadschneider, Phys. Rev. E **82**, 046111 (2010).

[10] D. Yanagisawa, A. Kimura, A. Tomoeda, R. Nishi, Y. Suma, K. Ohtsuka, and K. Nishinari, Phys. Rev. E **80**, 036110 (2009).

[11] F. Styns, L. van Noorden, D. Moelants, and M. Leman, Human Movement Sci. **26**, 769 (2007).

[12] D. Jezbera, D. Kordek, J. Kříž, P. Šeba, and P. Šroll, J. Stat. Mech. (2010) L01001.